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Evaluation of the Learning Process of Students Reinventing the General Law of Energy Conservation

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To investigate the relationship between context and concept we have constructed a conceptual learning path in which students reinvent the concept of energy conservation and embedded this path in two authentic practices.A comparison of the expected learning outcome with actual student output for the most important steps in the learning path gives us a detailed insight in how the teaching-learning sequence functions.The analysis identified several characteristics of authentic practices that enhance the learning process and showed that every step in this innovative approach is possible for students to take. In addition to learning the concept of energy conservation, embedding the learning process in authentic practices gives rise to a development of students' technological design and scientific skills.

Keywords: Authentic practices, context-based education, energy, guided reinvention, versatility, secondary education.

INTRODUCTION

In traditional education the concept of energy has been diagnosed as inflexible: research on Dutch exam students (Borsboom et al., 2008) and South African chemical engineering students (Liu et al., 2002) show that students have trouble in applying the concept of energy to various situations. Dutch chemistry students have been shown to have trouble in revising their ideas on energy when necessary (Kaper, 1997). By 'traditional' we mean the approaches as they are experienced by the majority of high school students in the countries of the cited sources. In such approaches energy is taught largely as a fact whose substantiation the students do not experience. We think that teaching the concept of energy as an unsubstantiated fact causes the trouble students have in applying and revising their ideas on energy.

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In traditional education mathematics is taught as a collection of indisputable facts as well. To resolve problems that stem from this and to substantiate the concepts that students are intended to learn, Freudenthal (1991) recommends the guided reinvention approach.

To make students see the relevance of science, Dutch curriculum innovation committees for the exact sciences have generally advised a context-based approach (Boersma et al., 2007; Commissie Vernieuwing Natuurkunde onderwijs havo/vwo, 2006). Prescriptions on how to implement contexts are not given explicitly so various ways to implement them are still open (Goedhart et al., 2001).

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State of the literature

- Although the concept of energy has many applications in technology and science, in traditional education students encounter difficulties when applying the concept in solving problems and revising their conception when necessary.
- The guided reinvention approach may be useful to substantiate the concepts that students are intended to learn and authentic practices as contexts may be useful to show students the relevance of what is learned.
- A difficulty in context-based education concerns the development of abstract concepts within contexts.

Contribution of this paper to the literature

- The method of design research has been employed to develop an innovative teaching-learning sequence in which students are intended to develop a versatile conception of energy conservation within authentic practices.
- A detailed description of the learning process within the developed teaching-learning sequence has been given by analyzing each learning step within it.
- In our research we have delivered a proof of principle that it is possible to develop an abstract concept such as energy conservation in authentic practices while developing students' technological design and scientific skills as well.

Gilbert categorizes the ways in which to implement contexts into four models (Gilbert, 2006). He chooses 'context as the social circumstances' as the most promising. In education based on such contexts teacher and students are seen as a community of practice. Boersma (2007) specifies such an implementation more precisely by choosing authentic practices as contexts.

Gilbert (2006) argues that the contextual problem should decide which concepts are useful in solving it. The problem posing approach states that learning should be driven by problems students can identify with (Lijnse & Klaassen, 2004). For every step in the learning process there should be a reason for the student to perform it. A combination of the problem posing approach with the use of authentic practices might be successful to show students the relevance of what is learned (Dierdorp et al., 2011; Westra, 2008; Bulte et al., 2006; Westbroek, 2005). In addition, Bulte recommends to have students make the procedures they use explicit, to aim the reflection phase at other typical problems within the same practice and to fit that reflection coherently into the scheme of activities (Bulte et al., 2006).

We have designed a context-based teaching-learning strategy in which students are intended to develop a versatile concept of energy conservation. In every step of the learning process we use authentic practices as contexts to involve the students in a meaningful activity in which the intended versatile concept can be developed. As Lijnse and Klaassen (2004) suggest we designed our teaching-learning strategy together with a scenario that predicts in detail the teaching–learning process as it is expected to take place. Our main focus now concerns whether it is possible for students to take each step in the teaching-learning process and thus guide them to reinvent the intended concept of energy conservation using the given assignments. We will illustrate this by a description of the learning process within the teaching-learning sequence. In our analysis attention will also be given to the interaction between the context and the conceptual development.

In the next section a summary of our final educational design is given. This is followed by a section on the Research Setup. In the Method & Results section a detailed scenario is given together with the results. The article ends with Conclusions and a Discussion of the findings.

EDUCATIONAL DESIGN

Conceptual development

We want students to reinvent the concept of energy conservation but we do not think students can do so in one go. Therefore we have designed three consecutive learning steps for students to take. In the first learning step students are to reinvent what we call partial laws of energy conservation, examples of which are given in Table 1. Reinventing a law involving more than two characteristic variables (terms) involves experiments which are difficult to perform.

In a second learning step, students need to learn the procedure for combining various partial laws into one law in order to expand the law of energy conservation to contain more than two characteristic variables. Each expansion requires a partial law that contains one characteristic variable already incorporated in the conservation law and at least one new characteristic variable. Since we think it is not possible for students to reinvent partial laws containing more than two characteristic variables this means that expanding the conservation law can only be done with one characteristic variable at a time (e.g. combining $\sum mgh + \sum mcT = k_6$ with $\sum mgh + \sum \frac{1}{2}mv^2 = k_7$ to form $\sum mgh + \sum mcT + \sum \frac{1}{2}mv^2 = k_8$).

In the case of the general law of energy conservation this process may continue for a long time but the end result may still be only a partial law of energy

Table 1. Examples of partial laws of energy conservation

Example situation from applicability domain	Examples of partial laws of energy conservation[a]
Lifting and lowering objects in balance.	$\sum mgh = k_1$
Insulated mixing of hot and cold substances (Lavoisier).	$\sum mcT = k_2$
Elastic collisions (Leibniz/Huygens).	$\sum mv^2 = k_3$
Frictionless object on a spring in a horizontal plane.	$\sum \frac{1}{2}mv^2 + \sum \frac{1}{2}Cu^2 = k_4$
Frictionless object on a spring.	$\sum mgh + \sum \frac{1}{2}mv^2 + \sum \frac{1}{2}Cu^2 = k_5$

The constants $k_1 \dots k_5$ in these partial laws of energy conservation are only constant under specific preconditions and may thus depend on ther variables and vary over different experiments.

Table 2. Learning trajectory

Conceptual learning step	Conceptual goal
I: Reinvent partial law of energy conservation.	e.g. $\sum mcT = k_2$
II: Combine partial laws of energy conservation.	e.g. $\sum mgh + \sum mcT = k_6$
III: Extrapolate the combination procedure through reflection.	$\sum mgh + \sum mcT + \sum \frac{1}{2}mv^2 + \cdots = k_7$[b]

Meant to describe the general law of energy conservation including any terms as yet unknown to students.

Table 3. Explicit contextual goals and hidden conceptual goals per assignment

Ass.	Students' contextual goal	Our conceptual goal
1	Design lifting apparatus.	$m_1h_1 + m_2h_2 = k_8$
2	Design a thermostatic mixer tap.	$m_1T_1 + m_2T_2 = k_2$
3	Design a rollercoaster.	$gh_1 + \frac{1}{2}v_2^2 = k_9$
4	Find experiments in which h increases and T decreases or the other way around. Find out whether a new law describing such experiments can describe all experiments so far.	$\sum mgh + \sum mcT = k_{10}$
5	Find out whether the law for the rollercoaster can be incorporated in the same manner.	$\sum mgh + \sum mcT + \sum \frac{1}{2}mv^2 = k_{11}$
6	Find out how many more terms can be added to the law.	$\sum mgh + \sum 426mcT + \sum \frac{1}{2}mv^2 + \sum \frac{1}{2}CU^2 + \cdots = k_{12}$[c]

Meant to describe the general law of energy conservation including any terms as yet unknown to students.

conservation albeit one that covers many situations. To reinvent the general law of energy conservation a third and final learning step is necessary. The students need to reflect on each step in the procedure of expanding the law of energy conservation to see whether those steps are always possible. Judging whether these steps are always possible cannot be done with complete certainty but those students that dare to take that risk have effectively reinvented the general law of energy conservation which is now applicable to any situation. The students that do not dare to take that risk may still have the intention to first try and see if an expansion is possible. Effectively this group has also accepted the general law of energy conservation. The three learning steps are summarized in Table 2.

In each of the three steps in this learning process the students have to adjust their conception to new information and after each successful revision the new conception has become more broadly applicable. Each step can thus be said to increase the versatility of the student's conception of energy.

To prepare the students for the reflection on the combination procedure we assume they need to have performed this procedure two times under the guidance of the teacher, the third time they may be able to perform it themselves and reflect on it. This means that at least four partial laws need to be reinvented.

Embedding in authentic practices

We want students to reinvent the general law of energy conservation while they are working, or learning to work, in a more or less authentic practice. Authentic practices in which physical laws are reinvented are technological design and scientific practices. We have chosen three technological design assignments in which students are to reinvent various partial laws of energy conservation and two scientific assignments in which

Table 4. The substeps in a technological design assignment together with the expected learning outcome (the substeps in bold are the ones investigated in this article)

Substep	Work phase	Expected student activity
1		Read introduction to the assignment.
2		Read description of the report structure.
3	Problem analysis	Find circumstances.
4		Find solutions to similar problems.
5		List tasks and requirements.
6	Problem definition	**Define problem accurately.**
7		**Use preconditions in problem definition.**
8	Cognitive modeling	List partial tasks.
9		Choose the four most important ones.
10		Find partial solutions to the most important tasks.
11		Combine partial solutions into a preliminary complete design.
12	Design proposal	**Formulate uncertainties.**
13		**Propose experiment to test uncertainties.**
14	Constructing a prototype	Construct prototype.
15		Answer uncertainties.
16		**Perform measurements.**
17		**Derive partial law from measurements.**
18		Write advice report.
19		Name partial law in advice report.
20		**Apply partial law in advice report.**
21		**Name preconditions on partial law in advice report.**
22	**Evaluation**	Describe best solution to assignment and future similar problems.
23		Rewrite law.
24		Check zero points and units for involved variables.
25		Expand to more objects than two.
26		Describe situations outside the domain of the law.
27		Choose most widely applicable, yet easiest usable notation.
28		Exercises on applicability.
29		Exercises on using the law.
30		Answers to exercises.

students are to combine those partial laws. In a third and final scientific practice the students are to reflect upon the combination procedure, resulting in what we call a metaconcept.

As an example of a technological design assignment the students were asked in the first assignment to come up with a design for lifting the heavy capstones on top of the pillars of the Parthenon in ancient Greece. In the first scientific assignment the students are asked, due to similarities between the two laws concerning height and temperature, to find a law that describes situations in which both height and temperature changes and whether such a law would describe the results of the earlier assignments correctly as well.

The six assignments each have a contextual goal which is given to the students as well as a conceptual goal which is not given to them. We present the two goals for each assignment in Table 3.

The three technological design assignments and three scientific assignments together result in a learning trajectory as shown in (fig. 1)where columns represent the three learning steps, and the six assignments are (part of) the rows. Assignment 3 has been moved one row down to be next to the corresponding combination assignment (assignment 3 next to assignment 5): the partial law for assignment 3 (describing the rollercoaster) is not combined with the two earlier reinvented partial laws (from assignments 1 and 2) until assignment 5.

We have divided the learning process within an assignment in work phases for technological design and for scientific assignments as described by Ellermeijer and De Beurs (Ellermeijer & de Beurs, 2004). In each work phase students are expected to take specific substeps towards both the contextual goal as well as our conceptual goal. The substeps and work phases are given in Table 4 for the technological design assignments and in Table 5 for the scientific assignments.

We consider the substeps and the evaluation phases given in bold in both types of assignment essential for

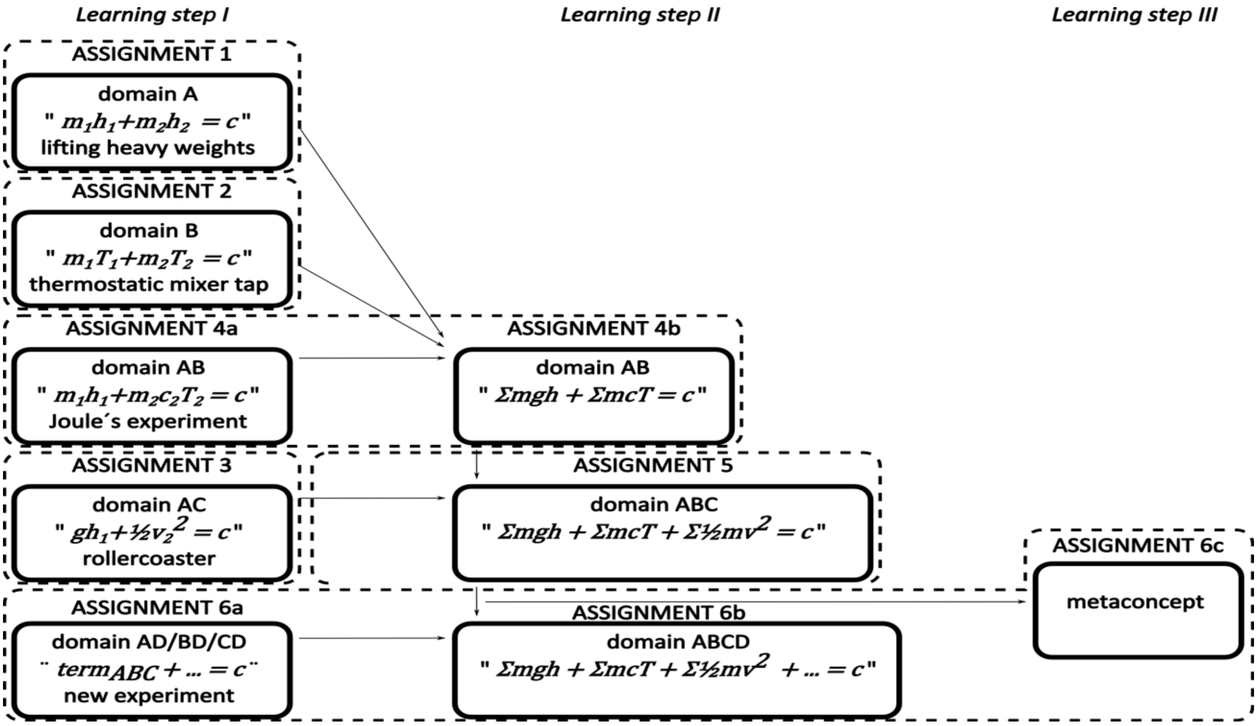


Figure 1. An overview of the six assignments combining the nine conceptual learning steps.

realizing the conceptual goal. These substeps will be analyzed in detail in Section 4.

Several substeps are only necessary for realizing the contextual goal (technological design substeps 1, 2, 3, 4, 5, 8, 9, 10, 11, 14, 15, 18, and 19 & scientific substeps 1, 2, 3, 5, 9, 12, 13, and 18) and are therefore left out of the analysis presented here. Other substeps are given to the students to train themselves (technological design substeps 28, 29, and 30 & scientific substeps 26, 27, and 28) or are taken in a classroom discussion led by the teacher (scientific substeps 13 to 17 but only during assignments 4 & 5) and are therefore also left out of the analysis. In both the technological design assignments and the scientific assignments the substeps are to be taken sequentially because they depend on the outcome of each previous substep.

More about the design process of the teaching-lerning strategy can be found elsewhere (Logman, Kaper & Ellermeijer, submitted-a).

RESEARCH SETUP

Research question

To identify the critical steps in the learning process we analyze the various work phases within the assignments. As described before, in each work phase students are expected to take specific substeps towards both the contextual goal and the conceptual goal. For each assignment our research question is:

- *To which extent do the various substeps of the teaching-learning sequence and the use of authentic practices contribute to the intended learning process?*

The contributions of the various substeps to the learning process will be analyzed based on the development of the concept of energy conservation, the use of authentic practices, and the development of technological design and scientific research competencies. A number of substeps in the teaching-learning strategy are repeated in subsequent assignments because to develop the concept of energy the corresponding conceptual learning step is repeated as a whole as well. It also offers us an opportunity to ascertain student's progress in taking similar substeps from assignment to assignment. Where applicable we will use Fisher's exact test as opposed to the Chi square test because of the small number of students involved in the try-out.

Experimental groups

The teaching-learning strategy aims at pre-university level sixteen- or seventeen-year-olds who have little or no quantitative knowledge about the concept of energy. During the third and final try-out the material was tested in four classes from three different schools. In school 1 the researcher himself taught a class of sixteen-year-olds. See Table 6 for an overview of the ages and number of students in each class.

Due to specific circumstances only 10% of the students in class 3 handed in their worksheets and final reports for assignment 6.

Table 5. The substeps in a scientific assignment together with the expected learning outcome (the substeps in bold are the ones investigated in this article)

Substep	Work phase	Expected student activity
1		Read introduction to the assignment.
2		Read description for the report structure.
3	Phenomenon analysis	State reason for performing the research.
4		**Describe phenomenon that connects two partial laws.**
5		Compare earlier assignments, experiments, and laws.
6		**Name preconditions of the partial laws.**
7		**Name domain of the partial laws.**
-	Problem definition	*Not applicable*
8	Cognitive modeling	**Describe experiment that combines the two laws.**
9		Roughly describe steps to be taken.
10	Experiment proposal	**Describe steps to derive a new partial law from measurements as precisely as possible.**
11		**Describe steps to combine partial laws of energy conservation as precisely as possible.**
12		Describe reasons behind every step[d]
13	Carrying out experiment	Watch demonstration, use earlier data, or read description.
14		**Describe measurements.**
15		**Derive partial law from data.**
16		**Start combining partial laws.**
17		**Find and name combined law.**
18		Write scientific report.
19		**Apply combined law in scientific report.**
20		**Name preconditions of combined law in scientific report.**
21		**Describe domain of combined law in scientific report.**
22		**Discuss combination procedural substeps in scientific report**[d]
23	**Evaluation**	Describe situations for uninvestigated expanded domain parts.
24		Rewrite law for each partial domain.
25		Reflect on which steps were taken as a preparation for future similar problems[e]
26		Exercises on applicability.
27		Exercises on using the law.
28		Answers to exercises.

[d] *In the last scientific assignment only.*
[e] *In the first two scientific assignments only.*

Table 6. An overview of the experimental classes

Class	School	Teacher	Age	Number of students
1	School 1	Researcher	16	6
2	School 2	Teacher 1	17	29
3	School 2	Teacher 2	17	30
4	School 3	Teacher 3	16	27

Instruments

Students worked mostly in couples. The main sources of information used for the analysis of the learning process are the handed in worksheets and reports. In addition we used observations by the researcher in the various classes.

METHOD & RESULTS

In this section we will discuss both method and results per substep. To answer the research question we will discuss to which extent the couples met our expectations per substep and per type of authentic practice.

For each substep we describe what we asked the couples to do (*task*), our criteria for judging their actions (*expectation*), our observations of what couples actually did (*result*), and our *conclusion*. Where relevant we will

Table 7. Percentage of couples taking the various substeps as intended for each technological design assignment and averaged over those three assignments

	Successful couples				Missing data[f]
Conceptual learning substep	**Ass. 1 (%)**	**Ass. 2 (%)**	**Ass. 3 (%)**	**Average[g] (%)**	**Average (%)**
6 Define problem accurately.	93.9	82.8	87.5	**89.2**	3.4
7 Use preconditions in problem definition.	*49.0*	*27.6*	*70.8*	***48.0***	*3.4*
12 Formulate uncertainties.	20.4	34.5	50.0	**31.4**	8.8
13 Propose experiment to test uncertainties.	26.5	51.7	62.5	**42.2**	8.8
16 Perform measurements.	22.4	51.7	37.5	**34.3**	13.7
17 Derive partial law from measurements.	8.2	27.6	25.0	**17.6**	13.7
20 Apply partial law in advice report.	28.6	31.0	45.8	**33.3**	13.7
21 *Name preconditions on partial law in advice report.*	*0.0*	*0.0*	*16.7*	***3.9***	*13.7*

[f] *The missing data is mostly due to couples from one teacher who treated students as responsible for their own learning process and therefore did not push them to hand in their worksheets.*

[g] *This average concerns a weighted average: assignment 1 was given to all the couples, while about half the couples were given assignment 2 and the other half were given assignment 3.*

Table 8. Main conclusions from the most essential substeps in the technological design assignments

Conceptual learning substep	**Main conclusion**
6 Define problem accurately.	This substep functioned as intended.
12 Formulate uncertainties.	Results are not yet good enough but the skill is not beyond students' capabilities.
13 Propose experiment to test uncertainties.	Progress indicates that even though it is a difficult skill for students they are capable of acquiring it.
16 Perform measurements. **17** Derive partial law from measurements.	Couples' skills of performing measurements and deriving a law from those measurements both showed considerable improvement.
20 Apply partial law in advice report.	Besides the couples that derived the law themselves there are other couples that applied the law in their advice reports.
7 Use preconditions in problem definition. **21** Name preconditions on partial law in advice report.	Naming the preconditions on the law is not essential in solving the given technological design problems so it is not a big issue for such assignments. However, they are essential when we intend to combine those partial laws later on during the scientific assignments. An increase is desirable.
- Evaluation phase in classroom discussion	The percentage of couples that are able to bring the intended law to the classroom discussion in the evaluation phase was enough to convince most of the other couples of the relevance of the law to the given problem.

give an *interpretation* of the results and *recommendations* for improving those results.

Technological design assignments

In the technological design assignments we expected the couples to learn how to take conceptual learning step I (reinvent a partial law of energy conservation) and experience the relevance of it to practical problems. In Table 4 we have divided this conceptual learning step into smaller substeps and indicated which of these are analyzed in detail.

Table 7 presents the results for the student couples per substep. Every couple worked on assignment 1, while about half the couples were given assignment 2 and the other half were given assignment 3. The first three columns show for each assignment the percentage of couples that based on our analysis took the corresponding substep in their worksheets the way we intended. The fourth column shows these numbers averaged over the three assignments. The last column shows the percentage of couples that did not hand in their worksheets or advice reports.

In the following section we will discuss each of these substeps. Some of the substeps have been grouped together because they describe similar substeps or interdependent substeps. In the order in which they are going to be discussed and based on the above

Table 9. The essence of the problem for each technological design assignment

Assignment 1	*How can we lift the capstone with as little force as possible?*
Assignment 2	*How can we match the real temperature to the scale on the tap?*
Assignment 3	*What is the highest point the rollercoaster may reach?*

percentages and that discussion the main conclusions are given in Table 8 as an overview.

Substep 6. define problem accurately

Task. The technological design problem was given to the couples in contextual terms: a client having a practical problem related to his line of business. The students were asked to analyze the problem further and describe the circumstances under which their design was to work, the tasks their design needs to perform and the requirements for their design. After this they were asked to define the problem more precisely and subdivide it into partial problems together with a preliminary solution. This is similar to authentic technological design practices in which initially the problem is not well-defined but has to become so in a first analysis.

Expectation. During their work on the problem the couples were expected to retrieve the essence of the problem, as presented in Table 9.

Result. During the problem analysis, problem definition, and cognitive modeling phase 89.2% of the couples addressed the essence of the problem. A positive example of a couple translated into English (from Dutch) is: *"Temperature needs to match numbers on the dial."*

The number of correct responses varied from 82.8% to 93.9% between the three assignments. The differences in the numbers were not significant..

The other couples (7.4%) did not formulate their requirements precise enough as illustrated in another quote: *"Put the temperature in a precise position."*

Conclusion technological design substep 6.
From the large majority of the couples being able to identify that part of the given problem that leads to our conceptual goal we conclude that this substep functioned as intended.

Substep 12: formulate uncertainties

Task. The students were asked to make their uncertainties about their preliminary solution explicit.

Expectation. We wanted the couples to sieve through the partial problems they had identified earlier and now identify those that according to them needed further research. In this process we expected the essence of the problem as described earlier in Table 9 to reappear.

Result. An average of 31.4% of the couples named the essence during this substep as an uncertainty in need of clearing up: *"How high is the rollercoaster supposed to be?"* This number rose significantly ($p<0.05$) from 20.4% for the first assignment to 34.5% or 50.0% for the next assignments 2 and 3 respectively, which were done parallel to one another.

A few couples came close but none of the other couples mentioned the essence of the problem according to our criteria even though they had not resolved the essence in their preliminary solution. Many of these couples mentioned uncertainties about other parts of the given problem which is illustrated by the following four uncertainties a couple had handed in:

"Does the counterweight work?
How do we attach the counterweight?
Is the construction stable enough?
Can we lift the capstone high?"

Interpretation. The large difference between the percentage of couples mentioning the essence of the problem in the earlier substeps (89.2%), and those mentioning it here as an uncertainty they would like to work on (31.4%) is remarkable. Technological design assignments offer students many partial problems from which it is difficult to select the ones most in need of further investigation. Because students are new to technological design this may explain their difficulties.

Conclusion technological design substep 12.
In the results we see an increase in the number of students that acquire the skill of selecting the most important partial problems. This suggests that this skill is not beyond students' capabilities. However the results are not yet good enough. Therefore we will analyze the next substeps on whether this low percentage hindered the learning process.

Substep 13: propose experiment to test uncertainties

Task. The students were asked to propose an experiment to answer their uncertainties.

Expectation. We expected the couples to describe an experiment for investigating the essence of the problem as described in Table 9. Such an experiment involves more than just a design test: it involves applied quantitative research.

Result. An average of 42.2% of the couples described an experiment that contained the essence of the problem. An example of a couples' description of such an experiment is the following: *"After that [experiment] we measure how high the car gets at certain velocities. Calculate, equation."*

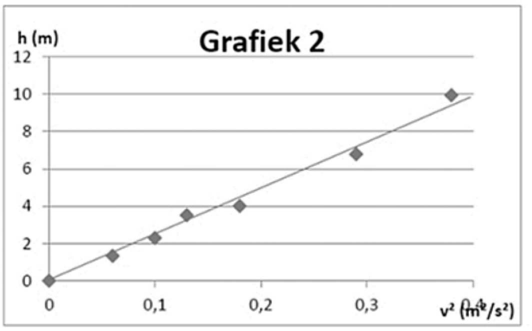
Table 10. The partial laws of energy conservation for each technological design assignment

Technological design assignment	**Intended partial law**[h]
Design lifting apparatus.	$m_1 \Delta h_1 = -m_2 \Delta h_2$
Design a thermostatic mixer tap.	$m_1 T_1 + m_2 T_2 = m_{total} T_{after}$
Design a rollercoaster.	$\frac{1}{2} v_{before}^2 = g h_{after}$

[h] *We also accepted any mathematical equivalent of these laws.*

Table 11. Example of procedural steps necessary to derive a law from measurements

	Procedural step to derive a partial law
1	Perform measurements.
2	Draw a graph from these measurements.
3	Linearize that graph when necessary.
4	Calculate the slope of the graph.
5	$\Delta y / \Delta x$ = the slope of the graph.



*"In grafiek 2 [translated: graph 2] one notices a straight line, therefore a linear relationship between h (m) and v^2 (m^2/s^2). Using this graph one can draft an equation (y=ax). First the slope needs to be calculated. Two points on the graph are needed to do so. We use (0,0) and (9.9,0.38). The slope may now be calculated: 0.38/9.9=0.038. In the equation that becomes h=0.038*v^2."*

Figure 2. Example of a part of one couple's advice report showing a correct derivation of a law governing the rollercoaster by calculating the slope of a $\boldsymbol{v^2}, \boldsymbol{h}$-graph.

Furthermore there is a significant upward trend ($p<0.005$) (from 26.5% to 51.7% or 62.5%) going from assignment 1 to either assignment 2 or 3.

In most other cases (33.3%) the couples merely wanted to test their prototype: "*Create a scaled construction and test it.*"

Answers in this category have in common that the couples are not aiming at a quantitative relationship yet. They are answering a yes/no type of question: will my solution work or not?

In a few cases (7.8%) couples focused on another subproblem (e.g. the influence of mass on the velocity of the rollercoaster car): "*We let the car run downhill with varying mass and measure the elapsed time.*"

The small remainder of the couples (7.8%) thought their solution was already complete so there was no need for an experiment.

Interpretation. The increase in expected answers from the previous substep (31.4%) to this one (42.2%) shows that couples did not write down all their uncertainties in the previous substep or that the teaching-learning sequence offers opportunities for students to trace their steps back to the intended learning path.

Conclusion technological design substep 13. *The results indicate that the intended transition from merely testing a scale model of the design (which does not solve the given problems completely) to investigating a quantitative relationship is a difficult one, though we found good progress (from 26.5% to 51.7% or 62.5%). The progress indicates that even though it is a difficult skill for students they are capable of acquiring it.*

Recommendation. In this try-out the teachers were already given the hint to ask the couples, when necessary, whether their experiment solves the given problem completely. We can now detail this hint further: (1) ask questions about the differences between the laboratory scale experiment and the real problem situation, (2) make students experience the need for a translation between the result of a scaled test and the real situation, and, if students do not arrive at this idea themselves, (3) suggest that a quantitative relation (e.g. a graph) could achieve this translation, but only if they dare to extrapolate results of the scaled experiment.

Table 12. Percentage of couples taking the various substeps as intended during the scientific assignments[i]

	Conceptual learning substep	Ass. 4 (%)	Ass. 5 (%)	Ass. 6 (%)	Missing data (%)[j]
4	Describe phenomenon that connects two partial laws.	72.0	72.0	57.1	20.4
6	Name preconditions of the partial laws.	74.0	80.0	69.4	20.4
7	Name domain of the partial laws.	64.0	66.0	75.5	20.4
8	Describe experiment that combines the two laws.	14.0	62.0	34.7	20.4
10	Describe steps to derive a new partial law.	40.0	66.0	75.5	22.4
11	Describe steps to combine partial laws.		28.0	18.4	22.4
14	Describe measurements.	56.0		59.2	32.7
15	Derive partial law from data.	44.0		51.0	32.7
16	Start combining partial laws.	64.0	58.0	44.0	38.8
17	Find and name combined law.	60.0	36.0	24.5	38.8
19	Apply combined law in scientific report.	22.0	16.0	6.1	42.9
20	Name preconditions of combined law in scientific report.	4.0	0.0	2.0	42.9
21	Describe domain of combined law in scientific report.	16.0	28.0	8.2	42.9
22	Discuss combination procedural sub-steps in scientific report.			28.6	42.9

[i] *For the empty cells in the table the corresponding substep was not taken during that assignment.*

[j] *The missing data is only given for assignment 6. Most of the missing data is caused by couples from one class that stopped handing in worksheets before or during assignment 6.*

Substeps 16 & 17 (conceptual learning step I): perform measurements & derive partial law from measurements

Because substep 17 can only be taken after substep 16 has been taken we discuss these two substeps together.

Task. During these substeps students were asked to perform their proposed experiment and write an advice report on their findings.

Expectation. No special worksheets were used at this stage. It was expected for couples to derive the partial law of energy conservation as mentioned in Table 10.

Only those couples were counted as having taken substep 17 in the case they showed a derivation of the partial law from their measurements (substep 16). Merely naming the partial law was not enough. We expected derivations in couples' advice reports to contain similar procedural steps to the ones mentioned in Table 11.

Result. Averaged over all three assignments 40.2% of the couples performed a quantitative experiment. The remainder of the couples continued their proposed plan of merely testing their preliminary solution.

An average of 34.3% of the couples showed correct measurements (substep 16). The couples that showed incorrect measurements may also have had an incomplete setup for their experiment. For example, a couple mixed 100 mL water of 15 °C and water of 60 °C resulting in mixed water with a temperature of 40 °C. Had they used 100 mL water of 60 °C the resulting temperature of 40 °C would have been incorrect (impossibly high). More likely their setup was incomplete because they had left out the amount of hot water perhaps assuming that the amount of hot water does not influence the resulting temperature: a known misconception (Meltzer, 2004; Erickson & Tiberghien, 1985).

An average of 17.6% of the couples showed a derivation that met our requirements (substep 17). An example of this is presented in figure 2.

From the first assignment to the second or third assignment students' skills improved significantly ($p<0.05$), both on performing measurements (22.4% to 51.7% or 37.5%) and deriving a law (8.2% to 27.6% or 25.0%). The relative percentage of couples deriving the intended law against the ones that showed correct measurements grew as well (from about one third to over half of the couples).

Conclusion technological design substeps 16 & 17. *Couples' skills of performing measurements and deriving a law from those measurements both showed considerable improvement. The skill of deriving a partial law will be further addressed in the scientific assignments. The percentage of couples that are able to bring the intended law to the classroom discussion in the evaluation phase needs to be enough to convince the other couples of the relevance of the law to the given problem. Whether the 17.6% is enough or not to contribute to the learning process will be decided based on the results of the evaluation phase.*

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Recommendation. For performing measurements the results for the third assignment lagged behind the results for the second assignment showing measuring velocity against height was more difficult than measuring amounts of water and temperatures. Therefore we recommend the teacher to help the students in measuring v,t- and h,t-diagrams and turning them into a v,h-diagram.

Substep 20: apply partial law in advice report

Task. At the end of each technological design assignment the couples were asked to write an advice report.

Expectation. We expected the students to apply their newly reinvented law in their advice report to calculate part of their solution. Those couples that used variables from the assignment together with the intended law to calculate an answer which was subsequently used to improve their advice report were classified as having taken this substep.

Result. Of all the couples 33.3% applied the law in their advice report:

- *"The afore-mentioned equation: c = h/v^2, if we fill in the measurements from our experiment we arrive at a constant C=0.04195. If we use this constant for a velocity of 180 km/h in other words 50 m/s we arrive at a maximal attainable height of 104.875 meter."*

There was a slight upward trend (from 28.6% to 31.0% or 45.8%)(not significant) going from assignment 1 to either 2 or 3. Nearly all couples that derived the partial law at an earlier stage applied it in their advice report.

Some couples merely mentioned the intended partial law of energy conservation (8.8% of all couples) as shown in the following example: *"To calculate the maximum height of a car one should use $\Delta(v^2) = 25\times h$."*

Most of the other couples (39.2%) did write advice reports containing several non-trivial aspects but did not address the essence of the problem as mentioned in Table 9. For example a final advice report on assignment 2 by a couple addressed the following aspects: the material for making the water tap, insulation around the tap so it does not become too hot, a lock at 39 °C, and a description of the mechanism to change temperature and water flow.

Interpretation. Of the couples 33.3% used the intended law in their advice reports. This shows that they understand the relevance of the partial law of energy conservation to the practical problem given and that they are capable of solving the essence of the given problem thereby raising the quality of their advice.

Additionally, 39.2% of the couples, while not using the partial law, still wrote non-trivial advice reports, thus a total of 72.5% of the couples wrote a non-trivial advice report. The non-trivial advice aspects other than using the intended law are evidence that technological design assignments offer a lot of distraction from a posed conceptual goal. Using these other aspects in an advice raises the quality of that advice and adds to the authenticity of students' activities.

Conclusion technological design substep 20. *Most students took the assignments seriously and were capable of giving useful advice on it. Besides the couples that derived the law themselves there are other couples that applied the law in their advice reports. Both will bring the law into the classroom discussion during the evaluation phase. This shows that the teaching-learning sequence offers opportunities for students to retrace their steps to the intended learning path by observing other couples at work.*

Substeps 7 & 21: name preconditions on partial law in advice report

Task. For substep 7 the couples were asked to describe the design requirements. For substep 21 the couples were asked to write an advice report.

Expectation. Substeps 7 and 21 are both about describing the preconditions on the reinvented partial law. At the time substep 7 is to be taken the partial law is not within the couple's view yet. The assignments are designed in such a way that the preconditions on the intended partial law can be identified by the couples as design requirements (e.g. as little friction as possible). In substep 7 we expected the couples to name these. For substep 21 we expected the couples to name the preconditions on the law in their advice reports.

Result. An average of 48.0% of the couples formulated the specific design requirements and thereby takes substep 7. The ones who did not formulate them still tended to conform to these requirements in their final solution, albeit without explicit reasoning. Describing the preconditions on the law in their advice report (substep 21) is only done by 3.9% of the couples.

Conclusion technological design substeps 7 & 21. *Naming the preconditions on the law is not essential in solving the given technological design problems so it is not a big issue for such assignments. However, they are essential when we intend to combine those partial laws later on during the scientific assignments. Therefore an increase in couples that see the relation between the design requirements and the preconditions on the law is desirable.*

Recommendation. *To increase the results for substep 21 similarities between the design requirements and the preconditions on the law should be given more attention in-between the two substeps and during the classroom discussion at the end of each assignment. This may for instance be done by adding a question to the classroom discussion which generalizes the situations in which the law cannot be applied by asking in which type of situations that is the case.*

Short description of the evaluation phase for technological design assignments

Task. The evaluation of each assignment was done in a classroom discussion led by the teacher. During this discussion the teacher remained in his role as a group leader of the design teams.

Expectation. First each couple describes their solution to the rest of the class. We expected rightly that at least one couple in each class would have been able to reinvent the intended partial law of energy conservation. After this the teacher asks the students which of the given solutions in their opinion is the best and why. Discussion is about what the optimal solution would look like. Here we expect the students to pick solutions in which the intended law was used because it improves the solution. Next follows a discussion on how the reinvented law could be rewritten to cover as many future design problems as possible. This way we intend to establish the domain of the law.

Conclusion technological design evaluation. The evaluation phase contributed effectively to the learning process. Many more students became aware of the intended law and its relevance to the advice report. The improvement from assignment 1 to assignment 2 or 3 in couples looking for a law can be seen as a result of this discussion phase. In this way the evaluation phase also contributed to diminishing the distraction of other non-trivial aspects

Result. In all of the discussions that the researcher observed (nine out of twelve in total) the students did choose the solutions in which the intended partial law of energy conservation was applied as the best and also added the non-trivial partial solutions to the ideal solution. Hereby the class as a whole showed that they had been interested in the assignment and wanted to contribute to improving the solution. As intended, they also appreciated the extra value that applying a law provided. During the rewriting of the law students preferred addition and multiplication over subtraction and division. This led to a notation of the law in which before and after terms ended up on either side of the equation in a natural way.

Scientific assignments

In the scientific assignments we expected the students to take conceptual learning steps II and III: combining partial laws of energy conservation and extrapolating that combination procedure. We have divided the conceptual learning steps into smaller substeps. In Table 5 we presented these substeps and indicated which of these are analyzed in detail.

The scientific assignments have an intrinsic order because each of them starts with the results of the previous assignment.

Because the teacher gave extra guidance (in the form of showing demonstrations and the mathematics behind combining partial laws) during assignments 4 and 5 it is not appropriate to average the percentage of successful student couples over all three scientific assignments. Instead of that we look upon couples' results for the substeps taken in assignments 4 and 5 as a preparation for taking those substeps by themselves in assignment 6. For each of the three assignments Table 12 presents the percentage of couples that took the corresponding substep based on our criteria for their worksheets and reports.

In the following section we will discuss each of these substeps. Some of the substeps have been grouped together because they describe similar substeps or interdependent substeps. In the order in which they are going to be discussed and based on the above percentages and that discussion the main conclusions are given in Table 13 as an overview.

Substep 4: describe phenomenon that connects two partial laws

Task. During assignment 4 the couples were asked to describe phenomena in which height decreases and temperature increases or vice versa to relate height to temperature. During assignment 5 they were asked to describe phenomena in which height or temperature decreases, and velocity increases or vice versa. During assignment 6 the couples were asked to describe a phenomenon in which any of the hitherto combined quantities (height, temperature, or velocity) decreases while a fourth new quantity increases, or vice versa.

Expectation. To expand the law phenomena are necessary that connect characteristic quantities. We expected couples to describe phenomena that connect the assignment-specific quantities mentioned above and take place in insulated systems. This criterion will identify the couples who understand that only such phenomena will lead to a new partial law of energy conservation.

Result. During assignments 4 and 5, 72.0% of the couples met our requirements as illustrated by the following examples:

- *Result for assignment 4: "A change of height causes heat through friction."*
- *Result for assignment 5: "One drops an object, it accelerates and loses height."*

The corresponding task in assignment 6 was more challenging (because couples had to identify a new quantity as well) but more than half the couples (57.1%) came up with phenomena involving one of the already established quantities and a new one. Even though interaction between couples was possible the couples were trying to identify a unique phenomenon. For example a couple mentioned that the amount of glucose in muscles decreases against an increase in height (lifting using muscles), temperature (friction by muscles), and

Table 13. Main conclusions from the most essential substeps in the scientific assignments

	Conceptual learning substep	Main conclusion
4	Describe phenomenon that connects two partial laws.	This substep contributes in the intended way to the learning process.
8	Describe experiment that combines the two laws.	The results show that this substep is learnable for students. The final results in assignment 6 are still low.
10	Describe steps to derive a new partial law.	This substep shows satisfying results. Students' skills in describing the necessary procedural steps for deriving a partial law are increasing considerably.
11	Describe steps to combine partial laws.	This substep showed promising results. However being able to discuss the complete procedure is essential in answering the final scientific problem so results for this substep need to be improved upon.
14 **15**	Describe measurements. Derive partial law from data.	The results for these substeps are encouraging. A (small) majority is able to derive a physical law from an experiment.
16 **17**	Start combining partial laws. Find and name combined law.	The promising results show that a quarter of the couples are able to derive a new partial law and to combine that new partial law into the already established law.
19	Apply combined law in scientific report.	The results for this substep are much too low and we need to reconsider our teaching-learning strategy.
6 **7** **20** **21**	Name preconditions of the partial laws. Name domain of the partial laws. Name preconditions of combined law in scientific report. Describe domain of combined law in scientific report.	The couples have learned how to describe the preconditions and domains of the partial laws during the technological design and scientific assignments to a good degree. In their scientific reports they fail to use the domain and therefore do not answer the given scientific problem completely.
22	Discuss combination procedural sub-steps in scientific report.	In assignment 6 half of all the couples by themselves concluded that it is always possible to expand the conservation law when necessary. The teaching-learning sequence at least partly fulfilled its goal. Improvement is desirable.
-	Evaluation phase in classroom discussion	The evaluation phase functioned in certain aspects but did not contribute enough to show students the relevance of each step in the combination procedure.

velocity (pushing using muscles). Further examples involved new quantities like the amount of fuel, the expansion of a spring, pressure, volume, etc.

- Approximately 22.5% of the couples gave answers that did not meet our criteria. They proposed phenomena that involved pairs of variables showing a cause-and-effect relationship, but in a non-insulated system. For example the following answer was given: "*A falling object, like a rock, will accelerate at the start of its fall but at a certain point this [the acceleration] will decrease while the velocity increases.*"

The supposed relationship is true when the object experiences friction. The couple has understood how to look for a law, and how to identify variables. However, the couple did not apply the idea of an insulated system, which is needed to find a (partial) energy conservation law.

Interpretation. In assignments 4 and 5 couples need to learn to describe a phenomenon that connects already established partial laws. In choosing such a phenomenon we also want them to realize the restriction (insulation) on the system that contains that phenomenon. The better results for assignments 4 and 5 compared to assignment 6 may be explained by the extra challenge inherent to assignment 6 to incorporate a previously uninvestigated quantity.

Conclusion scientific substep 4. The results show that it is possible for a majority of students to come up with suitable phenomena to investigate a possible new combination of partial laws. The diverse phenomena that couples come up with illustrate the possible width of the combined law that is sought. This substep therefore contributes in the intended way to the learning process.

Recommendation. Improvements in this substep may come from emphasizing that in all the earlier phenomena an increase of one variable simultaneously with a decrease of another took place in an insulated system. Therefore couples might expect they need these conditions if, like earlier, they want to find a law in which the sum of both variables is equal *at all times. Extra emphasis on these similarities may be given at various instants during all previous assignments. For instance, the teacher can ask which variable increases or decreases against the already known variable, or he can ask if the system is not influenced from outside (as in the case of the accelerating falling rock). In such a way quite a few couples were guided to discovering the role of glucose and fuels in certain phenomena.*

Substep 8: describe experiment that combines the two laws

Task. During this substep we asked couples to describe an experiment from which a suitable new partial law might be found.

Expectation. To expand the law an experiment is needed from which a new partial law of energy conservation can be derived. The couples were expected to state the quantities they would measure or vary in their experiment, which quantities they would keep constant and which objects would be involved. Only if the assignment-specific quantities are named (the same as in substep 4), and the objects involved are clear it is possible to assess whether as a whole the system was insulated. If that was the case the couples were qualified as successful in this substep. Experiments that are not insulated may lead to a physical law (e.g. a law that relates velocity to acceleration for an object falling through a medium) but they will not lead to a new partial law of energy conservation.

Result. Of the couples 14.0% came up with a suitable experiment during assignment 4. Of all the couples most came up with climbing a mountain and measuring height against temperature, as well as the (more or less insulated) object whose change of height causes the temperature change: the air. Only a few of these couples met our requirements and clearly stated the object of which they wanted to measure the temperature (in this example the object is the air): *"If you climb a mountain. The air grows colder when you go higher. Instead of climbing a mountain one may release an air balloon equipped with an altimeter and thermometer."*

Note that the object that changes height is "you" or "an air balloon" (not the air and not an insulated system).

In assignment 4 a majority of couples did not meet our requirements. In the following example the correct variables are named but the couple did not identify the air as the object involved: *"Lifting an object up a mountain, the height changes and the temperature as well."*

During assignment 5 the corresponding task showed significantly better results (62.0%)($p<0.0001$): most couples proposed the already performed rollercoaster experiment. Some couples proposed other experiments involving braking objects: *"A train that brakes. The track heats up due to friction. [Measure] the T of the wheels of the train. [Measure] the velocity of the train."*

In such cases the couples had difficulties in naming both the braking object and the brake or the surface as involved objects.

During assignment 6 the new experiment involved a new variable as well. Even though this means it may have become a more difficult task significantly more couples succeeded (34.7%)($p<0.05$) than during assignment 4. The following quote shows that the need for insulation has become clear to the couple:

- *"Heating water with a gas flame. We need a gas cylinder with a meter that tracks how much gas is burned. A thermometer to measure the temperature of the water before and after heating. [Keep constant] The phase of the water, the temperature of the surroundings of the water and the flame."*

A majority of couples did not meet our requirements. They either did not check whether other variables did indeed remain constant during their experiment or failed to describe the objects involved in the experiment. The next quote shows a couple that did not describe the objects that were involved in the experiment nor how they would keep other variables constant or how to exclude influences from the outside world: *"[Measure] spring elongation before-after, [Measure] height before-after, [Keep constant] things that you do not measure / cannot measure."*

Interpretation. Results for assignment 5 were higher than for the other two assignments because the earlier observed rollercoaster experiment is available as a suitable experiment. We notice an improvement in results from assignment 4 to 6 which shows that applying preconditions to proposed experiments is possible for students to learn. We also notice that many couples are focused on finding a quantitative law but not yet on a partial law of energy conservation. To find a partial law of energy conservation it is necessary to apply the precondition of having an insulated system and the couples need to identify the involved objects and describe how other variables are kept constant which they did not.

Conclusion scientific substep 8. *The results show that this substep is learnable for students. The final results in assignment 6 are still low. Not being able to apply the need for an insulated system appears to cause this result. This might give problems during the critical discussion on the general validity of the conservation law when discussing this substep.*

Recommendation. An extra question may be added to the scientific assignments after the couples have come up with an experiment. This extra question should ask the couples whether they are sure their experiment satisfies all the preconditions for finding a law as intended and whether they have described the relevant objects precisely enough. As this question is a difficult one the teacher must be ready to discuss students' experiment proposals with them.

Substep 10: describe steps to derive a new partial law

Task. In this substep we asked the couples to describe which procedure is necessary to find the new partial law following the experiment.

Expectation. This task is meant to reflect on earlier procedures concerning the derivation of a law and to prepare the couples for actually doing so. It is also necessary as a preparation for discussing whether this substep is always possible when discussing the general validity of the law at the end of assignment 6. The procedural steps that we expect the couples to mention concern the linearization of a graph of the measurements when necessary and the determination of the slope of that graph using $\frac{\Delta y}{\Delta x} = slope$.

Result. The number of couples that described the complete procedure necessary to derive such a law increased significantly from 40.0% in assignment 4 and 66.0% in assignment 5 ($p<0.05$). The onward rise to 75.5% in assignment 6 was not significant. An illustrative example from assignment 6 is the following: *"Draw a graph, create a straight line by using the square of one of the axes, calculate the slope, $\Delta y/\Delta x$ = slope"*

Conclusion scientific substep 10. In preparation for this derivation of a new partial law, substep 10 shows satisfying results. Therefore it appears to be a suitable moment to implement a procedure reflection as suggested by Bulte. Students' skills in describing the necessary procedural steps for deriving a partial law are increasing considerably, which we think will help in actually performing such a derivation later on in the learning process.

Recommendation. Results may improve if we tackle the problem of not mentioning the linearization of the data. After having seen the derivation of the partial law for the rollercoaster assignment in which a linearization is necessary it is possible to guide the couples to include a linearization of the graph in their description of how to derive a law from measurements. This can be done by asking what step is needed if the graph is not linear.

Couples that did not meet our requirements mostly forgot to mention the linearization of the measurements.

Interpretation. Being able to describe the procedure of deriving a partial law does not mean that couples are able to actually perform such a procedure. During the technological design assignments about a quarter of the couples showed they were capable of actually following this procedure during assignments 2 or 3 (27.6% or 25.0% respectively). Having been shown the procedure again during scientific assignment 4 the number of couples that were able to describe the procedure increased significantly to 75.5% ($p<0.0001$) in the final scientific assignment 6. We think that having performed the procedure themselves earlier helps in being able to describe it.

In her research of authentic practices Bulte (2006) recommends to create a need for making such procedures explicit. We have found such a need in the preparation for a scientific experiment.

Substep 11: describe steps to combine partial laws

Task. During this substep the students are asked to describe the steps taken during the previous assignment and which they think are necessary to combine the new law into the already established one.

Expectation. During this substep we wanted the couples to reflect on the combination procedure that was shown by the teacher in the previous scientific assignment. The aim of this reflection is to help couples in actually performing a combination of partial laws by themselves later on in the learning process. In the final assignment the couples also need to discuss every step in such a combination procedure to form an opinion on the general validity of the conservation law. The combination procedure consists of seven steps as summarized in Table 14.

Table 14. Procedural steps for combining partial laws[k]

	Procedural step
1	*Identify characteristic quantity.*
2	*Measure new quantity in relation to one of the already established quantities.*
3	*Establish relationship between those two quantities.*
4	*Rewrite the law into a notation in which before and after are moved to either side of the equation.*
5	*Multiply the whole equation by selected constants to make the term containing the already known quantity equal to a term appearing in the already established (now partial) law of energy conservation.*
6	*Add the term containing the new quantity to the already established law of energy conservation expanding it to include the phenomenon encountered.*
7	*Add sigma's to each term to generalize over more than one object for each type of term (i.e. for each form of energy).*

[k] We also accepted any equivalent of these procedural steps.

During assignment 4 this procedure was shown to the students by the teacher in a classroom discussion for the first time. Therefore a reflection on the procedure could not be asked from the couples at that stage. We made the first combination as easy as possible. At that stage two partial laws were available ($\sum mh = k_1$ and $\sum mcT = k_2$) and a third ($mh + mc^*T = k_6$) was established from Joule's experiment. During the classroom discussion little attention was given to the derivation of the latter law so we did not require the couples to state procedural steps 1, 2, and 3. Due to the partial laws that are available a combination of the laws is possible without taking procedural steps 4 and 5, making the procedure simpler.

Therefore during the reflection on the procedure in the subsequent assignment (assignment 5) we expected the couples to describe only procedural steps 4 and 7. During assignment 6 the complete procedure was expected and therefore our analysis focuses on that assignment.

Result. During assignment 6 18.4% of the couples were capable of describing a correct procedure to combine partial laws. An example of a successful couple's answer is the following:

- *"Remove delta-signs, fractions, brackets, and minus signs. Add summation signs. Look for similar term in both the old & the new law. Make the terms the same (by adding constants). Check if more terms can be made the same. Add the terms from the other law that are not yet mentioned in the new law: the two laws will become one law."*

Of all the couples 66.0% mentioned at least one procedural step. The least mentioned procedural steps were steps 1 and 5 (12.0% and 14.0% respectively). Almost half the couples mentioned procedural steps 2 and 3 (44.0% and 46.0% respectively) which relate to the earlier discussion of deriving a partial law in substep 10.

Interpretation. The simpler procedure reflection during assignment 5 showed 28.0% of all couples to meet our requirements. In assignment 6 not only the overall score was less (18.4%)(a non-significant decrease) the scores for the procedural steps 4 and 7 present in both reflections also decreased non-significantly. Having been shown how the procedure functions during assignments 4 and 5 it still proved challenging for the couples to remember and describe the steps that were taken. Even pointing students to their notes taken

Figure 3. A correct derivation of the partial law during assignment 6.

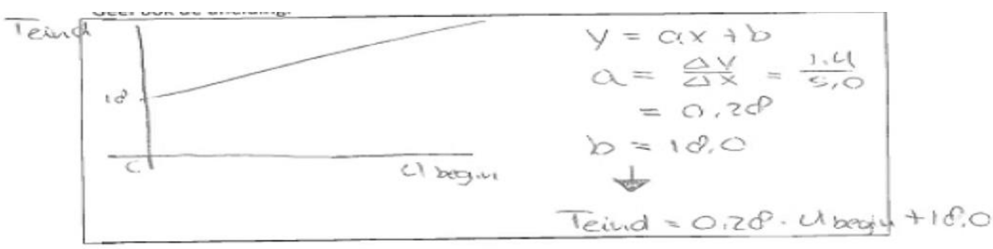

Figure 4. Result for assignment 6 of a couple assuming the law is linear.

during the previous combination and asking them to write down what had been done had many couples leaving out certain crucial steps.

Conclusion scientific substep 11. Having a majority of couples mentioning at least one of the procedural steps is promising. These students are better prepared for the discussion of the procedural steps during assignment 6 to establish the general validity of the conservation law. Having about one in six couples describe the complete procedure shows that it is possible for couples to take this substep. However being able to discuss the complete procedure is essential in answering the final scientific problem so results for this substep need to be improved upon.

Recommendation. Improvements in taking this substep may be achieved by showing the similarities between each combination more clearly by using a general combination method from the start of assignment 4. Besides that in assignment 6 we noticed that many couples stopped after describing the steps for deriving a partial law. Separating the derivation of a new partial law more clearly from the combination of that partial law into the earlier established law may make the need for both steps more clearly to the couples. This distinction may further be enhanced by making the couples realize that a new partial law does not yet increase the domain of the original law. Therefore we suggest the teacher to ask whether all previous experiments can now be described by the new law after each combination procedural step during the classroom discussion.

Substeps 14 & 15 (conceptual learning step I): describe measurements & derive partial law from data

Task. In these two substeps the couples were asked to derive a new partial law that describes certain measurements.

Expectation. Just like in the technological design assignment merely naming the partial law was not enough. We expected the students to show a derivation of the partial law including the measurements from which it was derived. Since showing such a derivation always includes a description of measurements we discuss these two substeps together.

Result. The number of couples that met our requirements increases significantly from 44.0% during assignment 4 to 51.0% in assignment 6 ($p<0.05$)(during assignment 5 the partial law for the rollercoaster was used and thus no new derivation was necessary). An example of a correct derivation is given in

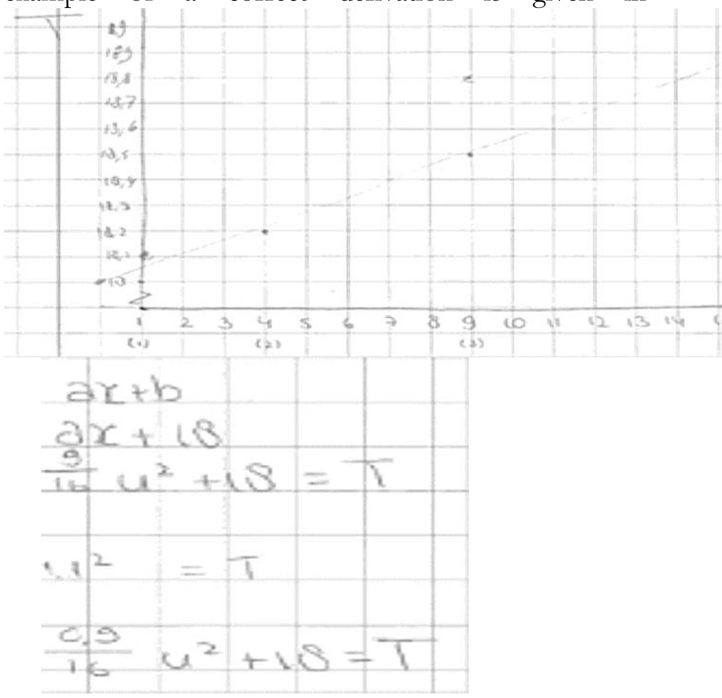

Figure 3.

A few of the couples (8.2%) that did not derive the correct law assumed the law would be linear (as shown in

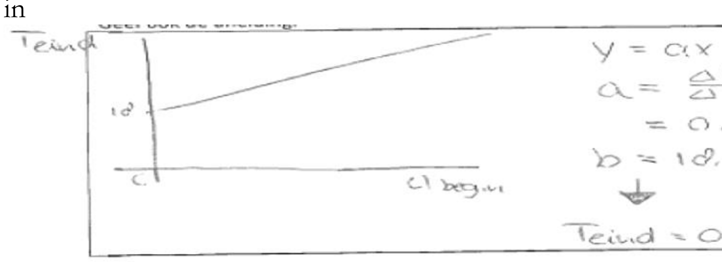

Figure 4) so they did describe correct measurements but arrived at an incorrect physical law.

These couples did not plot all the given data into their graph and did not check their resulting law against the given data either. Some more couples (another 8.2%) mentioned the data but could not derive a law from them. All other couples simply did not start work on these substeps.

Interpretation. About a quarter of the couples knew how to derive a law from measurements during the last two technological design assignments (27.6% and 25.0% respectively). In the scientific assignments this number steadily increases to 51.0% ($p<0.05$). The partial law they had to derive in assignment 6 was quadratic by nature and therefore comparable in difficulty to the most difficult relationship encountered so far (the quadratic relationship for the rollercoaster assignment). This comparable difficulty and the improvement of the results for this substep show that the scientific assignments help couples in acquiring the skill of deriving a partial law from measurements. The improvement of the results for this substep is accompanied by an improvement of the results for substep 10 which asks for a description of the procedure that needs to be performed here. We think that this reflection on the procedure after having tried to derive a partial law several times is key to the improvement in taking substep 15.

Conclusion scientific substeps 14 & 15. *The results for these substeps are encouraging. In trying to derive a partial law some students still assume a relationship to be linear. A (small) majority is able to derive a (quadratic) partial law of energy conservation. These couples are able to derive a physical law from an experiment. This is a necessary step when they are looking for a missing term in the conservation law. These couples are ready to take the next step in the overall learning process: combining partial laws of energy.*

Recommendation. As we have seen this problem of assuming a linear relationship during the technological design assignments as well we advise the teacher to insist on students drawing a diagram of the measured data and checking their resulting law against those measured data from that stage onward.

Substep 16 & 17: start combining partial laws & (conceptual learning step II) find and name combined law

Substeps 16 (start combining partial laws) and 17 (find and name combined law) are distinguished because we want to identify the students that started an attempt at combining both laws, as well as the ones who succeeded. Because substeps 16 and 17 concern the beginning and the end of the combination procedure we will discuss them together.

Task. The couples were asked to combine the new partial law into the earlier established one.

Expectation. For substep 16 we expected the couples to realize that deriving a new partial law of energy conservation does not mean that the law is already combined. Couples showed they realized this by continuing work on the partial laws of energy conservation in trying to combine them after having derived the new one. Once the couples had taken one of the procedural steps towards a combined law (see Table 14) they were counted as having successfully started work on combining partial laws.

$-\Delta(U_1^2)/\Delta T_2 = 18$
$-(U_{eind}^2-U_{begin}^2)/(T_{eind}-T_{begin})=18$
$-(U_{eind}^2-U_{begin}^2) = 18 \times (T_{eind}-T_{begin})$
$-U_{eind}^2+U_{begin}^2 = 18T_{eind} - 18T_{begin}$
$18T_{begin} + U_{begin}^2 = U_{eind}^2 + 18T_{eind}$
$T_{begin} + (1/18)U_{begin}^2 = T_{eind} + (1/18)U_{eind}^2$
$\sum(m\cdot c\cdot T)_{begin} + \sum(m\cdot c\cdot (1/18)U^2)_{begin} = \sum(m\cdot c\cdot T)_{eind} + \sum(m\cdot c\cdot (1/18)U^2)_{eind}$
Al gevonden formule:
$(\Sigma(m\cdot g\cdot h) + \Sigma(m\cdot c\cdot T) + \Sigma(½\cdot m\cdot v^2))_{na} = (\Sigma(m\cdot g\cdot h) + \Sigma(m\cdot c\cdot T) + \Sigma(½\cdot m\cdot v^2))_{voor}$
Nieuwe grootheid toegevoegd:
$(\Sigma(m\cdot g\cdot h) + \Sigma(m\cdot c\cdot T) + \Sigma(½\cdot m\cdot v^2) + \sum(m\cdot c\cdot (1/18)U^2))_{na} =$
$(\Sigma(m\cdot g\cdot h) + \Sigma(m\cdot c\cdot T) + \Sigma(½\cdot m\cdot v^2) + \sum(m\cdot c\cdot (1/18)U^2))_{voor}$

Figure 5 A couple combining partial laws correctly during assignment 6.[1]

In substep 17 (the final substep of conceptual learning step II) we expect students to successfully combine the partial laws and find the correct combined law including summation signs (Author, submitted-b).

Result. 44.0% of the couples started the combination procedure and 24.5% arrived at the correct combined law. An example is shown in

Of the couples 19.5% started the combination procedure but did not complete it successfully. Two thirds of these (12.2%) did find the right term to add to the earlier combined law so they came close to our requirement but did not write down the law that combines all previously reinvented partial laws:

- *"*$cmT + 0.056cmU_{before}^2 = cmT + 0.056cmU_{after}^2$*,*
- *substitute (*$0.056cmU^2$*):* $0.056 \cdot 850 \cdot 0.0050 \cdot U^2 = 0.238U^2$*,*

Note: 0.238 *is half of* 0.47 *(the C for the capacitor),*
so it becomes $\frac{1}{2}CU^2 = constant.$*"*

The remaining 7.3% that started combining did not get far. They did not get beyond the point in the procedure in which the partial law is rewritten in a notation such that the terms before and after the process are moved to either side of the equation: "$T_{before} + 0.06U_{before}^2 = 0.06U_{after}^2 + T_{after}$"

They did not generalize over multiple objects by adding a summation over each term and they did not combine all terms into one law.

The ones that did not start combining the partial laws got stuck during earlier substeps.

Interpretation. The percentage of couples that were successful (24.5%) is rather low but we have to consider that combining partial laws in assignment 6 could only start after students had successfully derived the new partial law. Only 51.0% of the couples had done this in substep 15. The main issue left is couples ending the combination procedure prematurely. Embedding the combination procedure in a scientific practice does not seem to be causing the low results. Many couples showed they are not able to relate steps of the combination procedure to changes in the law's domain: the use of summation signs and adding all terms together into one law are the only steps that influence the domain.

Conclusion scientific substeps 16 & 17. *The results show that a quarter of the couples are able to derive a new partial law and to combine that new partial law into the already established law. They have become capable of expanding the law of energy conservation when necessary. We interpret the result as promising because we have to take into account that the couples have to perform several sequential steps to arrive at a correct combined law. Much better results for two of the four classes (*Logman, Kaper &Ellermeijer*, submitted-b) indicate considerable improvement might be possible.*

Recommendation. The recommendations for these substeps are already given in the discussion of substep 11: describing the procedural steps to combine partial laws.

Substep 19: apply combined law in scientific report

Task. In the analysis for this substep we focus on assignment 6 because of the teacher guidance during assignments 4 and 5. In assignment 6 we asked the couples to find out how many more terms could be added to the law and to write a scientific report substantiating their answer.

Expectation. In their final scientific report we expected the couples to apply their newly reinvented combined law in answering the scientific problem given in the assignment.

Result. Of all couples 6.1% applied their newly reinvented combined law in their final scientific report to answer the given scientific problem. This is about a quarter of those couples that were capable of reinventing the combined law. An example of such an answer is the following:

- *"We have found a new term for the combination law and checked this against measurements.*
- *Our term* $= \frac{1}{2} C * U^2$
- *Undoubtedly, in the future more terms will be found which can be added to the combination equation.*
- *In principle infinitely many terms can be added."*

Other couples that had found the correct combined law merely mentioned the law but did not use it to formulate an answer to the given scientific problem. The majority of the couples had not reinvented the correct combined law.

Interpretation. During the earlier assignments 4 and 5 22.0% and 16.0% respectively applied the new combined law in their answer to the scientific problem. Especially in the final scientific assignment this substep is a necessary step to answer the given scientific problem. Even though applying the law is necessary to answer the contextual goal the number of couples that did so decreased significantly ($p<0.05$). Perhaps some couples did understand our conceptual goal but did not realize that in answering the contextual goal one more step was needed: answering whether the new combined law describes all previous experiments and in the case of assignment 6 answering how many terms can be added to the law. Another reason for the decrease in results might be that the teachers answered the scientific problems during the evaluation phase instead of discussing the answers from couples' scientific reports.

Conclusion scientific substep 19. The results for this substep are much too low and we need to reconsider our teaching-learning strategy. The assignment in itself may be too abstract: students lack to see what kind of answer they need to give.

Recommendation. To improve couples' understanding of the scientific assignments we suggest teachers to ask the couples to discuss their answers to the scientific problem as soon as a sizable number of couples have answered the scientific problem using the correct combined law. Alternatively the teacher may stimulate couples that found the correct combined law to help other couples in finding it as well. This way at least during assignments 4 and 5 it is possible for more students to come to know the correct combined law and form their own answer to the scientific problem based on that knowledge.

Substeps 6 & 7 and 20 & 21: name preconditions and describe domain of the combined law in advice report

As stated before the other substeps on preconditions and domains (substeps 6 and 7) are included in this discussion. The description of the domain of a law shows the possible situations or domain parts the law is able to describe, and the preconditions of a law focus on the limitations on situations in which that law is applicable.

Task. During substeps 6 and 7 the couples were asked to describe the limitations for the earlier established partial laws and to describe situations in which those laws are applicable. Substeps 20 and 21 are analyzed based on the couples' scientific reports.

Expectation. In substeps 6 and 20 we expected the couples to name preconditions such as: no friction, no outside influences or more generally insulated experiments or systems. In substeps 7 and 21 we expected the couples to describe the domain by naming which quantities are allowed to vary and which are to be kept constant. In substeps 6 and 7 we expected the couples to name the preconditions and domain of the earlier established partial laws whereas in substeps 20 and 21 we expected them to describe the newly combined law by its preconditions and domain.

Result. In the first scientific assignment, 4, 74.0% of the couples mention preconditions of the partial laws (substep 6) and 64.0% of the couples describe the domain of the partial laws (substep 7). In the subsequent assignments 5 and 6 these numbers remain high for substeps 6 and 7 (no significant changes).

Only 2.0% of the couples describe the preconditions of the combined law (substep 20) and 8.2% describe the domain of the combined law (substep 21) in their final scientific report. These numbers are low even during assignments 4 and 5 in which they are part of a classroom discussion.

Interpretation. During the preparation phase about three quarters of the couples mention the preconditions of the partial laws (substep 6) and about two thirds of the couples mention the domain (substep 7). However, in their scientific reports only a few couples mention the preconditions (substep 20) and domain (substep 21) of the combined law. Not mentioning the preconditions can be understood because the preconditions of the law are not necessary to answer the scientific problem (whether the newly combined law describes all the earlier partial laws it is combined from). On the other hand, to answer the given scientific problem a description of the domain is necessary.

How the resulting domain changes together with the preconditions of the combined law is key to understanding the extrapolation of the law to the general law of energy conservation during assignment 6 as well.

Conclusion scientific substeps 6 & 7 and 20 & 21. The couples have learned how to describe the preconditions and domains of the partial laws during the technological design and scientific assignments to a good degree. In their scientific reports they fail to use the domain and therefore do not answer the given scientific problem completely. The reason for this is comparable to the reason for the problems in substep 19.

Recommendation. The recommendations given under substep 19 are applicable here as well.

Substep 22 (conceptual learning step III): discuss combination procedural sub-steps in scientific report

Task. The couples are asked to write a scientific report to answer the final scientific problem of assignment 6.

Expectation. In their final scientific report we expected students to reflect on the combination procedure in order to form an opinion on the question whether it will always be possible to extend the law in the same way that it was possible up to now (conceptual learning step III). We expected the students to give a full discussion of all combination procedural steps (as mentioned earlier in Table 14) to substantiate their opinion.

Result. None of the couples gave a full discussion of all seven procedural steps. However several couples (28.6%) discussed one or more of the procedural steps. Most of these couples discussed only one or two procedural steps. Three couples (6.1%) discussed 6 out of the 7 necessary procedural steps. An example of such a couple's answer is the following:

- *"The characteristic quantity that increases or decreases in the experiment needs to have a certain relation to the term, together with any constants the relation to the other terms needs to become correct. Furthermore the new term needs to be written in before and after and in sigma notation. The terms need to contain quantities that are measurable and the units of the constants in the equation need to be correct."*

Of the seven necessary procedural steps for combining a new term into the conservation law the first procedural step was mentioned the most: 26.5% of all couples realized that a new characteristic quantity had to be related to an already established one. The final procedural step of adding summation signs to each term was only mentioned by one couple. Even though no couple mentioned all the procedural steps each of the seven procedural steps was mentioned by at least one couple.

In their final scientific report 46.9% of the couples stated they were convinced that a new form of energy could be added to the conservation law whenever such an addition is necessary (4.1% were hesitating in their statement). None of the couples explicitly stated that it would not be possible. Four couples (8.2%) did not formulate an answer to the question about the validity of the law.

Interpretation. Based on their concrete experiences having seen several expansions of the law most couples feel comfortable accepting the general validity of the conservation law. They do not see the need for a scientific proof by discussing all the procedural steps. Couples that did try to substantiate the law with evidence did not give a complete reasoning. The main cause for this is their difficulty with reflecting on which procedural steps are necessary to combine partial laws. They did not recognize taking more and different steps than predicted.

Conclusion scientific substep 22. In assignment 6 half of all the couples by themselves concluded that it is always possible to expand the conservation law when necessary. None of the couples stated that this would not be possible. Based on the results we can conclude that the teaching-learning sequence at least partly fulfilled its goal. Unfortunately the emphasis on analyzing the procedural steps did only function for about a quarter of the couples. The question whether this substep is appropriate for this level of students is difficult to answer. Because we expected it would contribute to anchoring the acceptance of the general law of energy conservation improvement is desirable.

Recommendation. If the choice is made to further develop the procedural reflection we suggest to finish assignment 6 with a scientific debate led by the teacher in which each of the procedural steps is discussed. During such a discussion the *need for discussing every procedural step should become clearer to the students.*

Short description of the evaluation phase for scientific assignments

Task. The evaluation of each assignment was done in a classroom discussion led by the teacher. During this discussion the teacher remains in his role as a group leader of the scientific teams. Because assignment 6 was the final scientific assignment of the teaching-learning sequence this assignment had no evaluation phase.

Expectation. First the students were asked to come up with situations from the uninvestigated domain parts. This was done to let students experience and establish the new expanded domain of the combined law. Subsequently the students were asked to rewrite the combined law as simple as possible for each of the partial domains. This results in the original partial laws and some new ones. These new laws show new possibilities provided by the combined law. In this way students experience the benefits of finding a combined law.

To prepare for future similar combinations, in a final question the students were asked to describe which procedural steps were taken to combine the partial laws. We expected this would prepare students for actually performing the process themselves during the final assignment.

Result. In all of the discussions that the researcher observed (six out of a total of eight) the students had little trouble in mentioning situations from the new domain parts. Rewriting the law posed problems to several students but the discussion with fellow students and the teacher helped them to find the correct partial laws for each domain part. The procedural steps were not discussed in this evaluation phase.

Conclusion scientific evaluation phase. The evaluation phase functioned regarding the aspects of giving concrete examples of situations from uninvestigated domain parts and simplifying the notation of partial laws where possible. The evaluation of the procedure did not contribute enough to show students the relevance of each step in the combination procedure.

Recommendation. The benefits of each step of the combination procedure need to be discussed more explicitly and the seven procedural steps for combining partial laws need to be incorporated in the teacher's manual so teachers can guide the reflection on those steps more clearly. A discussion of couples' answers to the scientific problems may be inserted at the beginning of this evaluation phase. This can clarify that naming the combined law is not enough to answer the given scientific problems completely.

CONCLUSIONS & RECOMMENDATIONS

Description of the learning process

The teaching-learning strategy consists of three technological design assignments followed by three scientific assignments which student couples work on. The only guidance by the teacher was given when students did not understand the contextual goal of the assignment. The conceptual goal the students had to find out by themselves. Because of the consecutive assignments students have several chances to retrace their steps to the intended learning path which we divided into several substeps. In this article we limit results to those substeps where steps towards the conceptual goal were expected (as opposed to steps towards the contextual goal).

The first substep in the technological design assignments to discuss was for students to define the technological design problem (e.g. designing a thermostatic water tap) more accurately (substep 6). In this step we expected students to at least identify that part of the problem which solution requires the intended partial law of energy conservation (e.g. the law about the final temperatures when portions of water with different temperatures are mixed). Most student couples identified the intended part of the problem as problematic in this substep which means that the assignments worked as expected. About the next substeps (12. select the most important partial problems, 13. investigate a quantitative relationship in testing a scale model of their design, 16. performing measurements and 17. deriving a physical law) we notice that many student couples did not take these expected steps in the first assignment but a significantly increasing number of couples managed to take these steps in subsequent assignments. This shows students' growth in both technological design skills and conceptual understanding. Writing up a useful advice posed little problems to the students. More student couples than the ones that derived a physical law used one in their advice report (substep 20) showing that writing an advice report provided students with a chance to notice the relevance of a physical law to a good design by watching other student couples using the law.

The improvement from the first assignment to the subsequent assignments in couples looking for a physical law can be seen as a result of the classroom discussion at the end of each assignment. The classroom discussion thus provided a clear opportunity for the students to notice the physical law and the relevance of it to their design and helped them thus to retrace their steps to the intended conceptual goal. In this sense the classroom discussion also helps to diminish the distraction from an intended conceptual goal that a technological design assignment has to offer. Substeps that call for improvement are mainly substeps 7 and 21, concerning the interaction between the design requirements and the preconditions on the physical law that describes that design. This is not as much a problem for the technological design assignments as it is for the subsequent scientific assignments in which these preconditions play an important role in combining partial laws of energy conservation.

In the first preparatory substep in the scientific assignments (4) most student couples were capable of identifying suitable phenomena to investigate a possible new combination of partial laws of energy conservation. After being shown a demonstration of an experiment and the derivation of the corresponding physical law students improve their results on describing experiments suitable for combining partial laws (substep 8) and on describing the steps needed to derive a physical law from measurements stemming from such experiments (substep 10). However, not many student couples manages to describe the complete procedure (see Table 14) for combining the relevant partial laws (substep 11). This may be caused by the expansion of the procedure from assignment 4 to 5. It is therefore recommended to make the two procedures as similar as possible.

More student couples were capable of actually deriving the intended partial law than before in the technological design assignments (scientific assignment substeps 14 & 15 as compared to technological design assignment substeps 16 & 17). About a quarter of the couples were able to derive a new partial law together with combining it into the already established law (scientific assignment substeps 16 & 17). These couples have become capable of expanding the law of energy conservation when necessary. We find this result promising taking into account that the couples have to perform several sequential steps to arrive at a correct combined law and it is only the second time we tried out to have students combine partial laws of energy conservation. Besides that, much better results for two of the four classes (Logman, Kaper &Ellermeijer, submitted-b) indicate that considerable improvement might be possible.

None of the student couples discussed the complete procedure (see Table 14) of combining partial laws to prove the general validity of the law (substep 22). Even though some student couples came close (they used six out of the seven necessary steps) the assignment may be too abstract for our level of students and different strategies are suggested. The problem concerning preconditions on, and the domain of, the combined physical law remained present in the scientific assignments as they were in the technological design assignments. This problem may be related to the problem of not being able to discuss the complete combination procedure and recommendations to resolve it have been given. The classroom discussion failed to show the students the relevance of each step in the combination procedure. More attention to these steps may be given during this discussion. Nonetheless about half the students were convinced that the law of energy conservation is generally valid at the end of the teaching-learning sequence. None of the students disagreed with this premise. In that sense the teaching-learning strategy partially fulfilled its goal.

For the most troublesome substeps in the technological design and all of the substeps of the scientific assignments recommendations have been given in the Results section to further improve the results of the teaching-learning strategy.

Short summative evaluation for conceptual learning steps I, II, and III

We have taken on the challenge of developing a teaching-learning sequence in which students are to reinvent the general law of energy conservation. We consider two types of students to have succeeded in this:

- *students that during the teaching-learning sequence were capable of substantiating the general law of energy conservation with evidence, and*
- *students that looked for a new term in the energy conservation law when a term appeared to be missing and were capable of adding such a new term.*

Applying these criteria we have observed that just over a quarter of the students (28.6%: substep 22 during the final scientific assignment) discussed part of the combination procedure to substantiate the general validity of the law of energy conservation with evidence (conceptual learning step III). Also about a quarter of the students (24.5%: substep 17 during the final scientific assignment) looked for a new term in the final scientific assignment and were capable of adding it to the law (conceptual learning step II). Almost half the students were explicitly convinced that the law of energy conservation is applicable to any situation. None of the students denied explicitly that the law would be applicable to any situation.

Before being able to expand the law of energy conservation one has to establish a partial law of energy conservation (conceptual learning step I). In the final assignment 51.0% of the students were capable of doing so.

The analysis of the afore-mentioned three conceptual learning steps shows the feasibility, even for sixteen-year-olds, of reinventing the general law of energy conservation during the teaching-learning sequence.

In the next sections we will discuss the learning process for the three learning steps in more detail. Subsequently we will discuss the influence of the chosen authentic practices on the learning process.

Conceptual learning step I

We expected the students to reinvent a partial law of energy conservation from measurements during assignments 1, 2, 3, 4, and 6. Students' skills in reinventing such a law grew considerably from about 8.2% (substep 17 during the first technological design assignment) to 51.0% (substep 15 during the final scientific assignment). The need for the derivation of such a law is more apparent to students in the scientific assignments than it is in the technological design assignments because in the latter many other aspects of the problem compete for the students' attention.

To assure that the reinvented partial law receives due attention in the technological design assignments each assignment was concluded by a classroom discussion in which all aspects that contribute to an optimal solution were evaluated. To let this happen spontaneously it is desirable that in every class at least one couple has applied the intended law of energy conservation in their advice report. In the technological design assignments about a third of our students did so (substep 20 in the technological design assignments). In all of the discussions that the researcher observed (evaluation phase of the technological design assignments) the students chose the solutions in which the intended partial law of energy conservation was applied as the best. This shows that students now see the relevance of these laws which we intended to achieve by embedding the teaching-learning sequence in technological design assignments.

Difficult steps for students were: the transition from merely testing a scale model of their design to investigating a quantitative relationship (technological design assignments, substeps 12 & 13), and deriving a relationship from data (technological design assignments, substep 17 & scientific assignments, substep 15). On both issues students developed their skills considerably.

Having been tried out three times conceptual learning step I functioned rather well. We did observe some couples having difficulty measuring height against velocity (substep 16 in Section 4.1) and deriving the quadratic law (substep 17 in Section 4.1) during the rollercoaster assignment. We suggest the teacher to help the couples once they have shown that they want to take these substeps. To prepare for conceptual learning steps II and III during the evaluation phase the similarity between the technological design requirements and the preconditions of the partial laws may be given more attention.

Conceptual learning step II

Partial laws of energy conservation were combined during assignments 4, 5, and 6. During assignments 4 and 5 the students performed the first steps themselves, up to and including proposing a suitable experiment for such a combination. Then during a classroom discussion the teacher showed the students how a new partial conservation law can be combined with the earlier established law. This teacher guidance during assignments 4 and 5 is intended to prepare the students for performing a combination themselves during assignment 6.

The majority of students were capable of identifying a phenomenon suitable for expanding the earlier combined law of energy conservation (substep 4 in the scientific assignments). Even though most of these students could describe the preconditions and the domain of this law (substeps 6 & 7 in the scientific assignments) only about a third of them applied the necessary preconditions in describing a suitable experiment (substep 8 in the scientific assignments). Students also had trouble recollecting the procedural steps (substep 11 in the scientific assignments) that were demonstrated earlier during assignments 4 and 5. The number of students that were capable of deriving a new partial law increased to over 50%. Almost half the students started combining the new partial law with the earlier established law (substep 16 in the scientific assignments) but only a quarter of them finished this procedure successfully and met our strict requirements (substep 17 in the scientific assignments).

The two main causes for the low results are that the role of preconditions to the reinvented laws has not been given enough attention in the learning process and that students had trouble in identifying which procedural steps were necessary for such combinations.
In the analysis of this try-out we have seen that it is in principle possible for students to take every substep for conceptual learning step II. We have also observed that a classroom discussion can be used to guide students through the process of combining partial laws (twice) to enable at least a part of the students to do it themselves a third time. However because only about a quarter of the students were capable of taking this conceptual learning step improvements are needed.

After having proposed an experiment that is suitable to investigate an expansion of the law the students may be asked to check their proposed experiment against earlier established preconditions by adding a question to the material and using teacher's guidance to discuss students' answers with them (substep 8 in scientific assignments).

Furthermore, we recommend to emphasize the changes in domain of the reinvented law by adding a question on its domain after each procedural combination step during the classroom discussion. This way it should become clear to the students how and when in the procedure the domain of the law really changes. This may help students see the need for every step in the combination procedure and make them continue beyond merely deriving a new partial law up to the point where the domain has actually been expanded (substep 17 and the evaluation phase of the scientific assignments).

Conceptual learning step III

This final conceptual learning step could only be taken by the students themselves during the final scientific assignment. Using a guided reinvention approach we want the students to substantiate the general validity of the law of energy conservation with evidence. To do so in our educational design a discussion of each step in the combination procedure is needed to assess whether an expansion of the law is always possible. A preparation for this step is done in the procedural reflections in assignments 4 and 5.

During conceptual learning step II only about a fifth of the students were capable of describing the necessary procedural steps for the combination (substep 11 in the final scientific assignment). Half of all the couples stated in their final report that the law of conservation is generally valid and none of the couples stated that this is not the case. About a quarter of the students substantiated the general validity of the law of energy conservation with evidence by discussing at least one of the necessary procedural steps (substep 22 in the final scientific assignment). However none of the students discussed all the necessary steps. The question whether delivering such a proof is appropriate for this level of students is difficult to answer.

The results may be improved by adding a classroom discussion after the students have formed their own opinion on the general validity of the law. In this classroom discussion the teacher needs to make sure that each of the combination procedural steps is critically discussed on whether it is always possible to take that step. This way more students may see the need of discussing the (complete) combination procedure in answering whether the law of energy conservation is generally valid. It will also help them in expressing their opinion on the general validity of the law of energy conservation more clearly in their final scientific report.

Authentic practices and skills

During the course of the first three assignments students' technological design skills improved. For the scientific skills it was harder to identify improvement because during the first two scientific assignments the teacher demonstrated most substeps. Therefore progress of the students on the corresponding scientific skills could not be assessed.

The students showed improvement in skills such as formulating uncertainties for their preliminary design, describing suitable experiments to test their design, performing measurements, deriving physical laws from data, and reflecting on the procedure of deriving such laws, as can be seen from the results for the corresponding substeps. In traditional teaching of the general law of energy conservation these skills normally are not addressed which implies that a separate treatment of technological designing, the scientific method, and the concept of energy conservation is not necessary, not even for an abstract concept such as energy conservation.

Elsewhere (Logman, Kaper & Ellermeijer, submitted-b) it is reported that in applying the concept of energy our students, one or two years before the final exam, scored similar results on the Energy Concept Inventory as Dutch exam students did in preliminary research. Prior to following our teaching-learning strategy our students had little or no quantitative knowledge about the concept of energy. They developed it in taking the conceptual learning steps as shown. This means that, by embedding the learning process in authentic practices, it does seem possible to develop physical concepts in students while enhancing their competencies as a physicist as well.

DISCUSSION

Limitations

The teaching-learning sequence takes on average 30 lessons of 50 minutes, which is about 30% more than a traditional approach. However this is mainly caused by the extra time necessary to introduce technological design and scientific practices. Although technological design assignments are part of the curriculum for some teachers this combination could be an obstacle.

The teaching-learning sequence has been limitedly tried out. Embedding conceptual learning steps II and III in a scientific practice has only been tried out in four classes during the final try-out. To achieve a more balanced teaching-learning sequence including a more elaborate teacher's manual at least one more round of try-out is necessary for these two steps. Conceptual learning step I embedded in technological design assignments has been tried out three times and in much more classes.

Teaching the concept of energy

There are many proposed approaches to teaching the concept of energy. Nearly all approaches state that energy is conserved as an indisputable fact except for two (Logman, Kaper &Ellermeijer, submitted-a; Genseberger & Lijnse, 2007). These two approaches and also the approaches of Falk and Lehavi (Lehavi et al., 2014; Falk et al., 1983) can be said to be phenomenological approaches. On purely phenomenological approaches some critical positions have been expressed by several researchers at the GIREP 2010 conference reported on by Stefanel (2014), describing it as a mission impossible. Considering our results we are confident to have shown that a predominantly phenomenological approach is possible. In its current, limitedly tried out, state our approach shows similar conceptual learning results as traditional approaches (Logman, Kaper &Ellermeijer, submitted-b) seemingly making the mission impossible possible. However, we are unsure whether we would call our approach purely phenomenological: an empirical philosophy of science is nowhere assumed.

Very few researchers report quantitative conceptual results on energy together with a description of their educational approach. Recently Solbes (2009) used exercise questions to provide such results and shows promising students' progress while making use of an approach based on conceptual change (Duit & Treagust, 2003), making such an approach an alternative that needs to be considered as well.

To compare conceptual results for various approaches there appears to be a need for a standardized conceptual test. We have used the qualitative Energy Concept Inventory that was in development by Swackhamer for such use (Swackhamer & Hestenes, 2005) but no recent progress seems to have been made. Prince and others have recently developed a more specific Heat and Energy Concept Inventory (Prince et al., 2012) but this inventory focuses mostly on heat.

A consistent learning trajectory on energy should continue beyond establishing the general law of energy conservation to include transformation, transfer, and degradation (Duit, 1986). In our teaching-learning sequence transformation and transfer have been addressed indirectly but degradation needs a further introduction. Besides that the use of forms of energy needs further elaboration as well. Several forms of energy are indistinguishable which should be made clear to the students in order to make them understand the concept of energy better. Even though we have ideas on these continuations more research on them is necessary. More research is also necessary to determine at which age it is most appropriate to expect students to take each of the increasingly difficult conceptual learning steps. Another aspect for further research is to establish more precisely at which point in the assignments the improvement in students' skills occurs and whether it facilitates or obstructs the intended conceptual development at that point.

Developing concepts in context-based education

Recent developments in the Netherlands in the teaching of biology and chemistry have been to implement contexts in the sense of authentic practices (Prins, 2010; Westbroek et al., 2009; Westra, 2008; Boersma et al., 2007). With the recent addition by Dierdorp (Dierdorp et al., 2011) for mathematics and our addition for physics it is clear that embedding a conceptual learning strategy in authentic practices is possible for all the exact sciences.

A problem in context-based education that we have tried to address concerns the development of abstract concepts in contexts (Westbroek et al., 2009; Parchmann et al., 2006; Pilot & Bulte, 2006; A. T. Schwartz, 2006). Some progress on this has been made by King and Ritchie (2013) who show that during a context-based course on water quality, fluid transitions between concept (chemistry test results) and context (a local creek and its surroundings) occur in students' statements. In our research we have delivered a proof of principle that it is possible to develop an abstract concept such as energy conservation in authentic practices. However, on the authenticity of the various assignments further research may be necessary.

REFERENCES

Boersma, K. T., van Graft, M., Harteveld, A., de Hullu, E., de Knecht van Eekelen, A., & Mazereeuw, M. (2007). *Leerlijn biologie van 4 tot 18 jaar*. Utrecht: CVBO.

Borsboom, J., Kaper, W. H., & Ellermeijer, A. L. (2008). The relation between context and concept in case of forming an energy concept. In *GIREP 2008: Physics Curriculum Design, Development and Validation*. Nicosia, Cyprus.

Bulte, A. M. W., Westbroek, H. B., de Jong, O., & Pilot, A. (2006). A research approach to designing chemistry education using authentic practices as contexts. *International Journal of Science Education*, *28*(9), 1063–1086.

Commissie Vernieuwing Natuurkunde onderwijs havo/vwo. (2006). *Natuurkunde leeft. Visie op natuurkunde in havo en vwo*. Amsterdam: Nederlandse Natuurkundige Vereniging.

Dierdorp, A., Bakker, A., Eijkelhof, H., & Van Maanen, J. (2011). Authentic practices as contexts for learning to draw inferences beyond correlated data. *Mathematical Thinking and Learning*, *13*(1-2), 132–151.

Duit, R. (1986). *Der energiebegriff im physikunterricht*. Kiel: Institut für die Pädagogik der Naturwissenschaften an der Universität Kiel.

Duit, R., & Treagust, D. F. (2003). Conceptual change: A powerful framework for improving science teaching and learning. *International Journal of Science Education*, *25*(6), 671–688.

Ellermeijer, A. L., & de Beurs, C. (2004). Technology enhanced physics education. In E. Mechlová (Ed.), *GIREP Conference 2004: Teaching and Learning Physics in new Contexts* (Vol. University, pp. 11–16).

Erickson, G., & Tiberghien, A. (1985). Heat and temperature. Part A: An overview of pupils' ideas; Part B: The development of ideas with teaching. In R. Driver, E. Guesne, & A. Tiberghien (Eds.), *Children's Ideas in Science* (pp. 53–84). Milton Keynes, UK: Open University Press.

Falk, G., Herrmann, F., & Schmid, G. B. (1983). Energy forms or energy carriers? *American Journal of Physics*, *51*(12), 1074–1077.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. (H. Freudenthal, Ed.). Dordrecht: Kluwer Academic.

Genseberger, R., & Lijnse, P. (2007). Bijlage bij: Op weg naar een wetenschappelijk energiebegrip. *NVOX : Tijdschrift Voor Natuurwetenschap Op School*, *32*(8), 364–365. Retrieved from http://www.nvon.nl/sites/nvon.nl/files/oud/nvox/nvox2007/supplementen/lessenserie_energie_1.pdf

Gilbert, J. K. (2006). On the nature of "context" in chemical education. *International Journal of Science Education*, *28*(9), 957–976.

Goedhart, M., Kaper, W. H., & Joling, E. (2001). Het gebruik van contexten in het natuurkunde- en scheikundeonderwijs. *Tijdschrift Voor Didactiek Der Beta-Wetenschappen*, *18*(2), 111–139; 111.

Kaper, W. H. (1997). *Thermodynamica leren onderwijzen. CD-beta reeks* (Vol. 27).

King, D. T., & Ritchie, S. M. (2013). Academic success in context-based chemistry: Demonstrating fluid transitions between concepts and context. *International Journal of Science Education*, *35*(7), 1159–1182.

Lehavi, Y., Eylon, B.-S., Hazan, A., Bamberger, Y., & Weizman, A. (2014). Focusing on changes in teaching about energy. In M. F. Tasar (Ed.), *Proceedings of The World Conference on Physics Education 2012, The Role of Context, Culture, and Representations in Physics Teaching and Learning2* (pp. 485–492). Istanbul, Turkey: Pegem Akademi.

Lijnse, P., & Klaassen, K. (2004). Didactical structures as an outcome of research on teaching-learning sequences? *International Journal of Science Education*, *26*(5), 537–554.

Liu, X., Ebenezer, J., & Fraser, D. M. (2002). Structural characteristics of university engineering students' conceptions of energy. *Journal of Research in Science Teaching*, *39*(5), 423–441.

Logman, P.S.W.M., Kaper, W.H., Ellermeijer, A.E. (submitted-a). An innovative educational approach aiming at a versatile concept of energy combining context-based education with guided reinvention.

Logman, P.S.W.M., Kaper, W.H., Ellermeijer, A.E. (submitted-b). Summative evaluation of a context-based approach making use of guided reinvention while aiming at a versatile concept of energy.

Meltzer, D. E. (2004). Investigation of students' reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course. *American Journal of Physics*, *72*(11), 1432–1446.

Parchmann, I., Gräsel, C., Baer, A., Nentwig, P., Demuth, R., & Ralle, B. (2006). "Chemie im kontext": A symbiotic implementation of a context-based teaching and learning approach. *International Journal of Science Education*, *28*(9), 1041–1062.

Pilot, A., & Bulte, A. M. W. (2006). The use of "contexts" as a challenge for the chemistry curriculum: Its successes and the need for further development and understanding. *International Journal of Science Education*, *28*(9), 1087–1112.

Prince, M., Vigeant, M., & Nottis, K. (2012). Development of the heat and energy concept inventory: Preliminary results on the prevalence and persistence of engineering students' misconceptions. *Journal of Engineering Education*, *101*(3), 412–438.

Prins, G. T. (2010). *Teaching and learning of modelling in chemistry education*. Utrecht, The Netherlands: CD-Bèta Press.

Schwartz, A. T. (2006). Contextualized chemistry education: The american experience. *International Journal of Science Education*, *28*(9), 977–998.

Solbes, J., Guisasola, J., & Tarín, F. (2009). Teaching energy conservation as a unifying principle in physics. *Journal of Science Education and Technology*, *18*(3), 265–274.

Stefanel, A. (2014). Discussion of strand on energy in upper secondary school. In M. F. Tasar (Ed.), *Proceedings of The World Conference on Physics Education 2012, The Role of Context, Culture, and Representations in Physics Teaching and Learning* (pp. 521–524). Istanbul, Turkey: Pegem Akademi.

Swackhamer, G., & Hestenes, D. (2005). *An energy concept inventory (draft version April 2005)*. Arizona State University.

Westbroek, H. B. (2005). *Characteristics of meaningful chemistry education*. Utrecht, The Netherlands: CD-Bèta Press.

Westbroek, H. B., Klaassen, K., Bulte, A., & Pilot, A. (2009). Providing students with a sense of purpose by adapting a professional practice. *International Journal of Science Education*, *32*(5), 603–627.

Westra, R. H. V. (2008). *Learning and teaching ecosystem behavior in secondary education*. Utrecht, The Netherlands: CD-Bèta Press.